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解答

1.

$$A|0\rangle = Z|0\rangle = |0\rangle, \quad A|1\rangle = Z|1\rangle = -|1\rangle, \tag{1}$$
$$\bar{A}|0\rangle = X|0\rangle = |1\rangle, \quad \bar{A}|1\rangle = X|1\rangle = |0\rangle, \tag{2}$$

$$\begin{aligned} B|0\rangle &= \frac{1}{\sqrt{2}}(Z-X)|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), \\ B|1\rangle &= \frac{1}{\sqrt{2}}(Z-X)|1\rangle = \frac{1}{\sqrt{2}}(-|1\rangle - |0\rangle), \end{aligned} \tag{3}$$

$$\begin{aligned} \bar{B}|0\rangle &= \frac{1}{\sqrt{2}}(Z+X)|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \\ \bar{B}|1\rangle &= \frac{1}{\sqrt{2}}(Z+X)|1\rangle = \frac{1}{\sqrt{2}}(-|1\rangle + |0\rangle) \end{aligned} \tag{4}$$

2.

$$\begin{aligned} \langle AB\rangle &= \langle\Psi^+|AB|\Psi^+\rangle \\ &= \frac{1}{2}(\langle 0|\langle 1| + \langle 1|\langle 0|)(A|0\rangle B|1\rangle + A|1\rangle B|0\rangle) \\ &= \frac{1}{2}(\langle 0|\langle 1| + \langle 1|\langle 0|)(|0\rangle\frac{1}{\sqrt{2}}(-|1\rangle - |0\rangle) - |1\rangle\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)) \\ &= -\frac{1}{\sqrt{2}} \end{aligned} \tag{5}$$

3.

$$\begin{aligned} \langle A\bar{B}\rangle &= \langle\Psi^+|A\bar{B}|\Psi^+\rangle \\ &= \frac{1}{2}(\langle 0|\langle 1| + \langle 1|\langle 0|)(A|0\rangle \bar{B}|1\rangle + A|1\rangle \bar{B}|0\rangle) \\ &= \frac{1}{2}(\langle 0|\langle 1| + \langle 1|\langle 0|)(|0\rangle\frac{1}{\sqrt{2}}(-|1\rangle + |0\rangle) - |1\rangle\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)) \\ &= -\frac{1}{\sqrt{2}}, \end{aligned} \tag{6}$$

$$\begin{aligned} \langle \bar{A}B\rangle &= \langle\Psi^+|\bar{A}B|\Psi^+\rangle \\ &= \frac{1}{2}(\langle 0|\langle 1| + \langle 1|\langle 0|)(\bar{A}|0\rangle B|1\rangle + \bar{A}|1\rangle B|0\rangle) \\ &= \frac{1}{2}(\langle 0|\langle 1| + \langle 1|\langle 0|)(|1\rangle\frac{1}{\sqrt{2}}(-|1\rangle - |0\rangle) + |0\rangle\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)) \\ &= -\frac{1}{\sqrt{2}} \end{aligned} \tag{7}$$

$$
\begin{aligned}
\langle \bar{A}\bar{B} \rangle &= \langle \Psi^+ | \bar{A}\bar{B} | \Psi^+ \rangle \\
&= \frac{1}{2}(\langle 0|\langle 1| + \langle 1|\langle 0|)(\bar{A}|0\rangle \bar{B}|1\rangle + \bar{A}|1\rangle \bar{B}|0\rangle) \\
&= \frac{1}{2}(\langle 0|\langle 1| + \langle 1|\langle 0|)(|1\rangle \frac{1}{\sqrt{2}}(-|1\rangle + |0\rangle) + |0\rangle \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)) \\
&= \frac{1}{\sqrt{2}} \qquad (8)
\end{aligned}
$$

4.

$$
\begin{aligned}
\mathcal{S} &= \langle AB \rangle + \langle A\bar{B} \rangle + \langle \bar{A}B \rangle - \langle \bar{A}\bar{B} \rangle \\
&= -2\sqrt{2} \qquad (9)
\end{aligned}
$$

上の式は、$|\Psi^+\rangle$ が古典論の満たすべき不等式を破っていることを示している。