



解答

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1.

$$\begin{aligned} |\langle 0|\psi \rangle|^2 - |\langle 1|\psi \rangle|^2 &= |(1, 0) \begin{pmatrix} \alpha \\ \beta \end{pmatrix}|^2 - |(0, 1) \begin{pmatrix} \alpha \\ \beta \end{pmatrix}|^2 \\ &= |\alpha|^2 - |\beta|^2 \end{aligned} \quad (1)$$

2.

$$\begin{aligned} |\langle +|\psi \rangle|^2 - |\langle -|\psi \rangle|^2 &= \left| \frac{1}{\sqrt{2}} (1, 1) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right|^2 - \left| \frac{1}{\sqrt{2}} (1, -1) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right|^2 \\ &= \frac{1}{2} (|\alpha + \beta|^2 - |\alpha - \beta|^2) \\ &= 2\text{Re}(\alpha\beta^*) \end{aligned} \quad (2)$$

3.

$$\begin{aligned} |\langle i|\psi \rangle|^2 - |\langle -i|\psi \rangle|^2 &= \left| \frac{1}{\sqrt{2}} (1, -i) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right|^2 - \left| \frac{1}{\sqrt{2}} (1, i) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right|^2 \\ &= \frac{1}{2} (|\alpha - i\beta|^2 - |\alpha + i\beta|^2) \\ &= -2\text{Im}(\alpha\beta^*) \end{aligned} \quad (3)$$